The annuity puzzle remains a puzzle [☆]

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ABSTRACT

We examine incomplete annuity menus, background risk, bequest motives, and default risk as possible drivers of divergence from full annuitization. Contrary to what is often suggested in the literature, we find that full annuitization remains optimal if saving is possible after retirement. This holds irrespective of whether real or only nominal annuities are available. Whenever liquidity is desired, individuals save sizeable amounts out of their annuity income to smooth consumption shocks. Similarly, adding equity-linked annuities to the menu does not increase welfare significantly, since individuals can invest in stocks in order to get the desired equity exposure.

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1. Introduction

In this paper we model optimal decumulation of retirement wealth. Prior research has shown that, in simple stylized settings, full annuitization of available wealth upon retirement is optimal for individuals who only face uncertainty about their time of death. Yaari (1965) shows that risk-averse agents with intertemporally separable utility who are only exposed to longevity risk, and with no desire to leave a bequest, find it optimal to hold their entire wealth in annuities if these are actuarially fair. This argument is extended by Davidoff et al. (2005) to cases with more risk factors and more general utility functions. Full annuitization is optimal in these models since the annuities generate a mortality credit that cannot be captured otherwise. We explore the implications of incomplete annuity markets and background risk on optimal annuity demand, where in contrast to the previous literature we do not model background risk as health cost risk since this risk is low in Europe. Furthermore, we calculate upper bounds for the bequest motive and default risk of the annuity provider for which full annuitization remains optimal.

In the literature the policy recommendation that all pension wealth should be annuitized has been challenged. These papers are partly motivated by the observation that very few individuals voluntarily purchase annuity products when they reach the retirement age (Bütler and Teppa, 2007; Mitchell et al., 1999). This empirical fact is often referred to as *the annuity puzzle*. In this paper we focus on four of the main factors that have been put forward to challenge the claim that full annuitization is optimal: incompleteness of the annuity market, background risk, bequest motives, and default risk. The first factor in our analysis is that annuity menus are typically incomplete. In many cases only nominal annuities are available rather than annuities which hedge inflation risk or which give exposure to equity markets. So if only nominal annuities are sold, agents still incur inflation risk and, on top of that, the nominal income in real terms is decreasing with age while agents prefer a flat consumption pattern. Such incomplete annuity menus have been found to result in large welfare costs (Horneff et al., 2008a; Koijen et al., 2011)[1]. The second factor emphasizes that annuities are irreversible due to adverse selection and people face borrowing constraints. This implies that annuities cannot be sold or borrowed against if liquidity is needed, for instance in case of breakdown of a durable consumption good or health costs. In contrast to previous work (Peijnenburg et al., 2016; Turra and Mitchell, 2008; Pang and Warshawsky, 2010), we do not focus on health costs, but background risk more generally. In Europe out-of-pocket health costs are negligible and we explore whether non-health cost background risk (a non-autocorrelated process) influences annuity demand. We find that background risk that is not autocorrelated reduces annuity demand only slightly at most. The third and fourth factor that we analyse are bequest motives and default risk of the annuity provider. Specifically, we calculate upper bounds for a bequest motive and default risk of the annuity provider to explore whether this will reduce the annuity demand. However, we find that for reasonable parameters, again, full annuitization remains optimal. The reason is always the same: agents can profit from the mortality credit and save out of annuity income to rebuild a liquid wealth buffer. Note that this single buffer serves multiple purposes: it allows smoothing background risk shocks, it allows profiting from the equity premium, and it can be left as bequest.

We analyze a comprehensive stochastic life cycle model from retirement onwards. An individual optimally allocates a fraction of wealth to an annuity at age 65. Every period an agent decides how much to consume and to save, and how to allocate liquid wealth between stocks and a riskless bond. Unlike other research, we explicitly allow the agent to save again out of annuity income. The model includes important risks a retiree faces, namely longevity risk, background risk, inflation risk, and capital market risk. Recently developed numerical methods are used to solve the model.

We find that (almost) full annuitization is optimal irrespective of whether real or only nominal annuities are available. Neither incomplete annuity markets nor background risk lead to a sizeable reduction of optimal annuitization levels. Individuals allocate approximately their entire wealth to annuities and save out of their annuity income to insure against shocks. If background risk hits them the saved liquid wealth is used as a buffer and consumption is temporarily reduced to rebuild the buffer. So incomplete annuity markets do reduce utility levels, but not the demand for annuities.

During retirement agents accumulate a sizeable amount of liquid wealth. The median savings account is at its maximum (in real terms) around age 84 and amounts to approximately 25% of initial wealth at age 65. Saving during retirement is driven by four factors: (1) redistribution of consumption to later periods when the real value of the nominal annuity income is low. Furthermore people save to hedge against (2) inflation risk and (3) background risk. Finally, wealth accumulation allows people to benefit from (4) the equity premium. We disentangle these four reasons and find that, the anticipatory motive to save (1) is most important. Furthermore inflation risk induces a large amount of precautionary savings; it increases the amount accumulated in the savings account by 50%. Expenses due to background risk are a substantial reason for saving, but less so than inflation risk. The possibility to gain equity exposure does not increase savings significantly.

Furthermore we find that adding variable annuities to the menu does not increase welfare significantly. This result contrasts the findings in Koijen et al. (2011) and Brown et al. (1999), because we assume that agents can invest in equity during retirement. In aforementioned papers, investment in equity, other than via the variable annuity, is not allowed during the retirement period. Hence the only manner to get the equity premium is via the variable annuity, which results in higher welfare gains from variable annuities, compared to our case where agents can invest directly in equity during retirement.

Similar to our paper Davidoff et al. (2005) also examine the effect of incomplete annuity markets on annuity demand. They find that the low annuity purchases in reality can only be reconciled by a large mismatch between the desired consumption path and available annuity income paths. In their paper they determine the optimal demand for a real annuity, when the optimal real consumption pattern is not flat. They assume a habit formation utility function, which creates the mismatch between the desired real consumption path (U-shaped or upward sloping) and available income path (flat). While incomplete annuity markets do explain the lack of full annuitization, they cannot explain the low levels of annuitization found in reality. Our paper examines a similar question but approaches it from a different angle. We assume a desire for a smooth consumption path in real terms and show that, even if only nominal annuities are available, full annuitization is still optimal.

We assume annuitization should take place once, at retirement. Several arguments can be given to motivate this choice. First of all in several countries the decision whether to annuitize your pension account or take a lump sum is, due to the tax legislation, to take place at retirement. Furthermore mandatory annuitization of a fraction of wealth at younger ages reduces adverse selection costs that are generated when the annuity date can be chosen. Rothschild (2009) uses a long time series to

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[1] Which does not necessarily means that optimal annuity levels are reduced, as we will find later.

test for adverse selection in annuity markets and finds significant selection effects. These adverse selection costs are ignored in most papers. A third reason for our assumption of a single conversion opportunity at retirement is that in reality people make financial decisions very infrequently rather than annually. Finally Agarwal et al. (2009) show that the capability of individuals to make financial decisions declines dramatically at higher ages, hence it seems optimal to make these decisions at younger ages when a person is still able to do so.

Even in case gradual annuitization is possible, most conclusions of this paper will not be affected: background risk and incomplete annuity markets are generally not enough to explain less than full annuitization. Clearly, restricting annuitization to take place at retirement only reduces total annuitization over the life cycle, as after a certain age full annuitization will be optimal due to the increasing mortality credit. As a result, without this assumption the annuity puzzle is even stronger.

In this paper we largely ignore a number of other potential drivers of annuity demand. These include the presence of loads in annuity prices (see for instance Mitchell et al., 1999), private information on health status (Turra et al., 2008), high means-tested benefits (Bütler et al., 2016), high pre-annuitized wealth levels (Dushi and Webb, 2004), minimum annuity purchase requirements Pashchenko, 2013), and family composition (Brown and Poterba, 2000; Kotlikoff and Spivak, 1981). A number of papers have analyzed annuity demand from a US perspective where health expenses are in general only partially covered by insurance policies. Ameriks et al. (2011) and Peijnenburg et al. (2016) find that out of pocket medical expenses reduce the optimal annuity demand. Furthermore, several behavioral explanations have been put forward, for example framing of the annuity choice (Agnew et al., 2008; Brown et al., 2008), mental accounting (Hu and Scott, 2007), and complexity of the annuity product (Brown et al., 2013).

The remainder of the paper is organized as follows. Section 2 describes the individual's preferences, the financial market, the benchmark parameters, and the numerical method to solve the dynamic programming problem. Section 3 contains detailed results for the benchmark case. Robustness checks are subsequently performed in Section 4. Section 5 concludes.

2. The retirement phase life cycle model

2.1. Individual preferences and constraints

We consider a life-cycle investor during retirement with age $t \in 1, ..., T$, where $t=1$ is the retirement age and T is the maximum age possible[2]. The individual's preferences are presented by a time-separable, constant relative risk aversion utility function over real consumption, C_t. More formally, the objective of the retiree is to maximize

$$V = E_0 \left[\sum_{t=1}^{T} \beta^{t-1} \left(\prod_{s=1}^{t} p_s \right) \frac{C_t^{1-\gamma}}{1-\gamma} \right], \tag{1}$$

where β is the time preference discount factor, γ denotes the level of risk aversion, and C_t is the real amount of wealth consumed at the beginning of period t. The probability of surviving to age t, conditional on having lived to period $t-1$, is indicated by p_t. We denote the nominal consumption as $\overline{C_t} = C_t \Pi_t$, where Π_t is the price index at time t.

The individual invests a fraction w_t in equity, which yields a gross nominal return R_{t+1} in year $t+1$. The remainder of liquid wealth is invested in a default-free bond and the return on this bond is denoted by R_t^f. The intertemporal budget constraint of the individual is, in nominal terms, thus equal to

$$W_{t+1} = (W_t + Y_t - B_t - \overline{C_t})(1 + R_t^f + (R_{t+1} - R_t^f)w_t), \tag{2}$$

where W_t is the amount of financial wealth at time t, Y_t is the annual nominal annuity income, and the expenses due to background risk are indicated by B_t. The timing of decisions is as follows. At retirement the agent decides which fraction of wealth to invest in annuities[3]. Subsequently, the individual receives annuity income and incurs expenses due to background risk. After this exogenous shock, the agent decides how much to consume and subsequently invests the remaining liquid wealth. In case the annuity income plus wealth at the beginning of the period is insufficient to pay the expenses and consume, the individual receives a subsistence consumption level. In our benchmark specification this happens in 0.02% of the cases, i.e., this subsistence level is so low that agents prefer to avoid it. The decision frequency for the optimal consumption and asset allocation is annually.

The individual faces a number of constraints on the consumption and investment decisions. First, we assume that the retiree faces borrowing and short-sales constraints

$$w_t \geq 0 \quad \text{and} \quad \iota' w_t \leq 1. \tag{3}$$

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[2] We assume a fixed retirement age and do not explore optimal annuitization and retirement timing as in Milevsky and Young (2007).

[3] As discussed before, limiting the annuity decision to take place at retirement in effect tests whether this additional constraint reduces optimal annuity demand. However, as we will present later, even if agents are limited to buy annuities at retirement, it is still optimal for agents to fully annuitize. If we would allow agents to annuitize at a later age, this result would not change and agents would annuitize 100% of wealth. The reason is that the mortality credit is higher at more advanced ages, hence it is even more advantageous to buy annuities then.

Second, we impose that the investor is liquidity constrained

$$\overline{C_t} \leq W_t, \tag{4}$$

which implies that the individual cannot borrow against future annuity income to increase consumption today.

2.2. Financial market

The asset menu of an investor consists of a default-free one-year nominal bond and a risky stock. The nominal interest rate, stock returns, expected inflation, and actual inflation are modeled as a VAR. The return on the stock is normally distributed with an annual mean nominal return μ_R and a standard deviation σ_R. The returns to equity are uncorrelated to inflation and the interest rate. The nominal interest rate at time $t+1$ equals

$$r_{t+1} = r_t + a_r(r_t - \mu_r) + \epsilon^r_{t+1}, \tag{5}$$

where r_t is the instantaneous nominal short rate and a_r indicates the mean reversion coefficient. μ_r is the long-run mean of the nominal instantaneous short rate, and ϵ^r_t is normally distributed with a zero mean and standard deviation σ_r. The yield on a default-free bond with maturity h is a function of the instantaneous short rate in the following manner:

$$R^{f(h)}_t = -\frac{1}{h}\log(A(h)) + \frac{1}{h}B(h)r_t, \tag{6}$$

where $A(h)$ and $B(h)$ are scalar functions and h is the maturity of the bond. The real yield is equal to the nominal yield minus expected inflation and an inflation risk premium. Hence the real yield, nominal yield, and expected inflation are time-varying, the inflation risk premium is fixed.

We model inflation, because we are interested in optimal annuitization levels in a world with inflation, but where only nominal annuities are available. For the instantaneous *expected* inflation rate we assume

$$\pi_{t+1} = \pi_t + a_\pi(\pi_t - \mu_\pi) + \epsilon^\pi_{t+1}, \tag{7}$$

where a_π is the mean reversion parameter, μ_π is long run expected inflation, and the error term $\epsilon^\pi_t \sim N(0, \sigma^2_\pi)$. Subsequently the price index Π follows from

$$\Pi_{t+1} = \Pi_t \exp(\pi_{t+1} + \epsilon^\Pi_{t+1}), \tag{8}$$

where $\epsilon^\Pi_t \sim N(0, \sigma^2_\Pi)$ are the innovations to the price index. We assume there is a positive relation between the expected inflation and the instantaneous short interest rate, that is the correlation coefficient between ϵ^r_t and ϵ^π_t is positive. The parameters we use are described in Section 2.3.

We consider single-premium immediate life-contingent annuities with real or nominal payouts. Consequently, the annuity income is given by

$$Y = PR_0 A^{-1}, \tag{9}$$

where PR_0 is the premium and A is the annuity factor. The single premium is equal to the present value of expected benefits paid to the annuitant and we assume an actuarially fair annuity[4]. The annuity factor, A, is thus equal to

$$A = \sum_{t=1}^{T} \exp\left(-tR^{(t)}_0\right) \prod_{s=1}^{t} p_s, \tag{10}$$

where $R^{(t)}_0$ is the time zero yield on a zero coupon bond maturing at time t. The interest rate term structure that is applied is either nominal or real depending on the type of annuity. We study in Section 4 the effect of loads on the annuitization decision.

The annuity factor for a variable annuity payout is similar to Eq. (10), but $R^{(t)}_0$ is equal to the assumed interest rate (AIR), which is fixed. The annual annuity income depends on the return of the portfolio backing the annuity, R^A_t, and is equal to

$$Y_t = PR_0 A^{-1} \prod_{t=1}^{T} \left(\frac{1+R^A_t}{1+AIR}\right). \tag{11}$$

The AIR determines whether, in expectation, the annuity payout stream increases or decreases over time. The annuity income is constant over time in case the AIR is equal to the return of the underlying portfolio, R^A_t. If the AIR is below R^A_t, then the nominal income stream is upwards sloping over time.

In Fig. 1 we display the mean annuity income in real terms for various types of annuities. Naturally the real income stream from the real annuity (solid line) is flat, and throughout this paper we normalize this to unity. This way of normalization allows for a simple comparison of various strategies. Furthermore, we see that the real income stream from the nominal annuity is decreasing over time, which is the dashed-dotted line. Early in retirement the real income generated from the nominal annuity is higher than from the real annuity. The income from the nominal annuity in real terms decreases over time from about 1.4 to 0.5. In addition we see that payout pattern of the variable annuity is largely influenced

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[4] We assume implicitly that survival probabilities are known and that there is no uncertainty regarding future survival probabilities. Bayraktar et al. (2009) explore the impact of uncertain future survival probabilities on the pricing of annuity products.
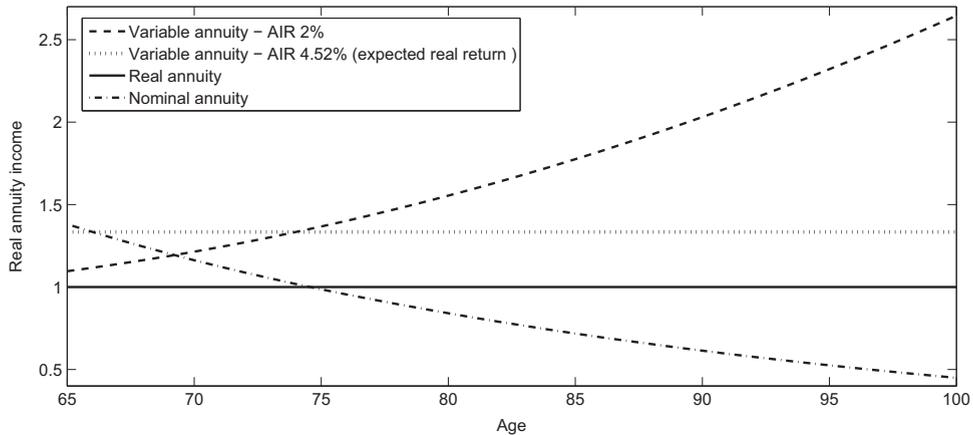
Fig. 1. The annuity income levels in real terms for various types of annuities The figure displays the (expected) annuity income over the life cycle in real terms generated by four types of annuities. We use the average inflation rate, average interest rate, survival probabilities, and equity premium to calculate these expected trajectories. We display the real income from a nominal, real and variable annuity. In case of the variable annuity we show the results for an assumed interest rate of 2% and 4.52%. The latter AIR equals the expected nominal return on the portfolio backing the annuity minus expected inflation.

by the AIR. When the AIR equals the expected return on the portfolio backing the annuity minus the expected inflation, the expected annuity income in real terms is flat. If we look at the dashed line which is the income pattern from a variable annuity with an AIR of 2%, we see in expectation an increasing income in real terms.

We assume that the expenses due to background risk are lognormally distributed with an annual mean μ_B and a standard deviation σ_B. Furthermore we assume that these expenses do not exhibit autocorrelation.

2.3. Benchmark parameters

The previous sections present the specification of the life-cycle preferences and the financial market. In this section, we set the parameter values for the benchmark case. In Section 4 we provide robustness analysis with regard to the most important parameters. In accordance with Pang and Warshawsky (2010) and Yogo (2012) we set β, the time-preference discount factor, equal to 0.96. The risk aversion coefficient γ is assumed equal to 5 for ease of comparison, since this is equivalent to Pang and Warshawsky (2010) and close to the parameter choice of Yogo (2012) and Ameriks et al. (2011). Initial wealth is such that, if the individual would annuitize fully in real annuities, the (real) income for the rest of the lifespan equals unity. We call this real annuity income (i.e., when all wealth is invested in a real annuity) the Full Real Annuity Income (FRAI). The mean expenses due to background risk are 10% of the FRAI, with a standard deviation of 7%. Furthermore we choose a subsistence consumption level of about 25% of the FRAI[5].

The equity return is normally distributed with a mean annual equity premium of 4% and an annual standard deviation of 20%, which are close to the parameters in Cocco et al. (2005). The mean instantaneous short rate is set equal to 4%, the standard deviation to 1%, and the mean reversion parameter to -0.15. The inflation risk premium is 0.5%, so that the time-varying real short rate is $r_t^{(r)} = r_t - \pi_t - 0.5\%$. The correlation between the instantaneous short rate with the expected inflation is 0.40. The parameters on the inflation dynamics are taken from Koijen et al. (2010). They find a mean inflation of 3.48%, a standard deviation of the instantaneous inflation rate of 1.38%, a standard deviation of the price index of 1.3%, and a mean reversion coefficient equals -0.165. The assumed interest rate for variable annuities is equal to 4%, which is similar to Horneff et al. (2009) and Koijen et al. (2011)[6]. The portfolio linked to variable annuity consists 100% of equity. Furthermore we will perform robustness checks to assess whether the results hold for different values for the individual preference parameters and financial market parameters. Time ranges from $t=1$ to time T, which corresponds to age 65 and 100 respectively. The survival probabilities are the current male survival probabilities in the US and are obtained from the Human Mortality Database[7]. We assume a certain death at age 100. The average male life expectancy, at age 65, is 81. We will perform robustness analysis using female survival probabilities and the average female life expectancy is 84.

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[5] The dollar equivalents of these numbers are as follows. Median wealth at age 65 is $335,000, which is the total of non-annuitized and annuitized wealth for a single, estimated in Pang and Warshawsky (2010). We also perform the analysis for other wealth levels. The annuity income if the entire wealth is invested in a real annuity is $22,645 (which is then normalized to unity). The subsistence consumption level is $6000, which is close to the consumption floor estimated in Ameriks et al. (2011). The mean expenses due to background risk are about $2250 and the standard deviation is $1600.

[6] The US National Association of Insurance Commissionaires requires that the AIR may not be higher than 5%. Furthermore Horneff et al. (2009) remark that 4% is commonly used in the US insurance industry.

[7] We refer for further information to the website, www.mortality.org.
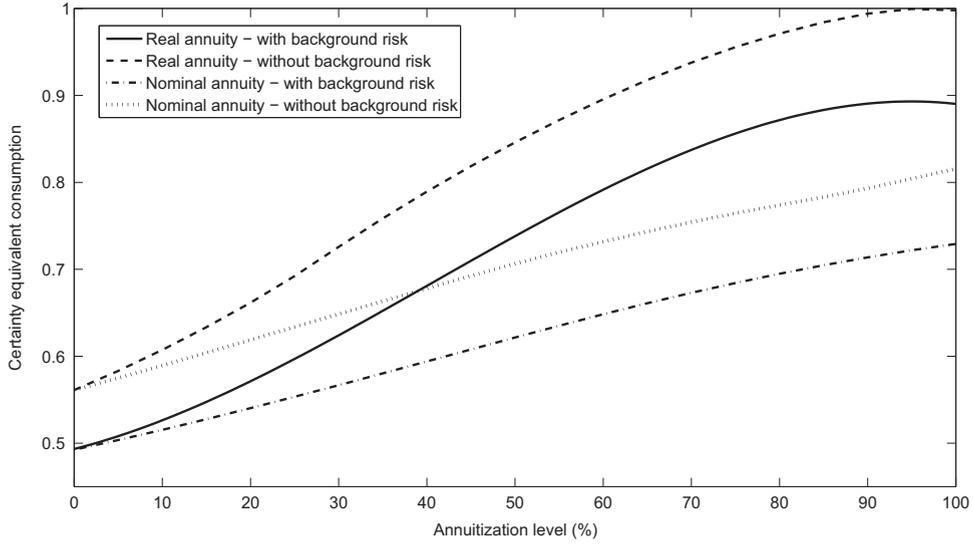
Fig. 2. Optimal annuitization levels results for the benchmark model. The figure displays the certainty equivalent consumption for the life-cycle model with and without background risk and nominal or real annuity income. Post-retirement equity investment of liquid wealth is included in the model. The optimal annuitization strategy is the level that generates the highest certainty equivalent consumption. All numbers are relative to the FRAI, which is the real annuity income if 100% is invested in a real annuity.

2.4. Numerical method for solving the life cycle problem

Due to the richness and complexity of the model it cannot be solved analytically and, hence, we employ numerical techniques instead. We use the method proposed by Brandt et al. (2005) and Carroll (2006) with several extensions added by Koijen et al. (2010). Brandt et al. (2005) adopt a simulation-based method which can deal with many exogenous state variables X_t, with, in our case (R_t^f, π_t). Wealth acts as an endogenous state variable. For this reason, following Carroll (2006), we specify a grid for wealth *after* (annuity) income, expenses due to background risk, and consumption. As a result, it is not required to do numerical rootfinding for the optimal consumption decision.

The optimization problem is solved via dynamic programming and we proceed backwards to find the optimal investment and consumption strategy. In the last period the individual consumes all wealth available. The value function at time T equals:

$$J_T\left(W_T, R_T^f, \pi_T\right) = \frac{W_T^{1-\gamma}}{1-\gamma}. \tag{12}$$

At all other points in time, the value function satisfies the Bellman equation

$$V_t\left(W_t, R_t^f, \pi_t\right) = \max_{w_t, C_t}\left(\frac{C_t^{1-\gamma}}{1-\gamma} + \beta p_{t+1} E_t\left(V_{t+1}\left(W_{t+1}, R_{t+1}^f, \pi_{t+1}\right)\right)\right). \tag{13}$$

For each period we find the optimal asset weights by setting the first-order condition equal to zero

$$E_t\left(C_{t+1}^{*-\gamma}(R_{t+1} - R_t^f)/\Pi_{t+1}\right) = 0, \tag{14}$$

where C_{t+1}^* denotes the optimal real consumption level. Because we solve the optimization problem via backwards recursion we know C_{t+1}^* at time $t+1$. Furthermore we simulate the exogenous state variables for $N = 1000$ trajectories and T time periods hence we can calculate the realizations of the Euler conditions, $C_{t+1}^{*-\gamma}(R_{t+1} - R_t^f)/\Pi_{t+1}$. We regress these realizations on a polynomial expansion in the state variables to obtain an approximation of the Euler condition

$$E\left(C_{t+1}^{*-\gamma}(R_{t+1} - R_t^f)/\Pi_{t+1}\right) \simeq \tilde{X}_p'\theta_h. \tag{15}$$

In addition we employ a further extension introduced in Koijen et al. (2010). They found that the regression coefficients θ_h are smooth functions of the asset weights and, consequently, we approximate the regression coefficients θ_h by writing them as polynomial expansion in the asset weights:

$$\theta_h' \simeq g(w)\psi. \tag{16}$$

The Euler condition must be set to zero to find the optimal asset weights

$$\tilde{X}_p'\psi g(w)' = 0. \tag{17}$$

For each period we find the optimal consumption by solving the following first-order condition:

$$C_t^{*-\gamma} = \beta p_{t+1} E_t \left(\frac{\Pi_t}{\Pi_{t+1}} C_{t+1}^{*-\gamma} R_{t+1}^{P*} \right). \tag{18}$$

3. Results on optimal annuitization and savings

In Sections 3.1–3.6, we focus on the optimal allocation to nominal and real annuities and in Section 3.7 we determine the welfare gains of adding variable annuities to the annuity menu.

3.1. Optimal annuitization strategies at retirement when facing background risk and incomplete annuity menus

As shown by Davidoff et al. (2005) full annuitization is optimal if the annuity market is complete. However, this might not be the case if no annuity is available which offers equity exposure or provides inflation protection and/or the agent is exposed to background risk. Fig. 2 presents the certainty equivalent consumption for various levels of annuitization, conditional on optimal consumption and asset allocation strategies. In all cases (almost) full annuitization is optimal. Hence, optimal annuity demand is not lowered, even though annuity markets are incomplete and agents face background risk.

The welfare gains over no annuitization are substantial. For instance, in case real annuities are available, but there is no background risk, full annuitization leads to an increase in annual certainty equivalent consumption from 57% of the FRAI to 100% of the FRAI[8]. If no annuities are available, welfare is thus reduced by about 43%. The magnitude of these welfare gains are in line with the findings in Davidoff et al. (2005) and Mitchell et al. (1999). For many individuals part of their wealth will be annuitized for institutional reasons, for example in the form of social benefit payments or Defined Benefit pensions. The results show that, also in that case, an increase in the level of annuitization from say 50% to 100% brings about a very substantial welfare gain.

Markets may be considered to be even more incomplete when only nominal annuities are available. Individuals might be induced to decrease annuity demand to protect against inflation risk and to shift income in early retirement to later years when the real value of the annuity income is lower. The dotted line displays the certainty equivalent consumption when an agent can only buy a nominal annuity and does not face background risk. Again we find that full annuitization is optimal. This implies that the fact that the annuity market is incomplete does not have a material impact on the optimal annuitization level, given that we allow saving from annuity income.

The optimal annuity demand is also hardly affected by the presence of background risk. The solid line in Fig. 2 shows that (almost) full annuitization is remains optimal. Obviously, background risk reduces the attainable utility levels, but the curves are still essentially increasing: more annuitization leads to more utility. Later we will see that the main difference with the case without background risk is that the agent accumulates wealth out of the annuity income to cover shocks in background risk and plans consumption to rebuild these buffers when needed.

Pang and Warshawsky (2010) find that in a life-cycle model with health costs as background risk, annuity demand actually increases. The reason for this contrasting result is that they do not model annuitization as a one-time decision that needs to be made at retirement age, but optimize annually over the equity-bond-annuity portfolio. In effect, the annuitization decision is modeled as a repeated portfolio allocation decision. Health costs are an additional risk factor which drives households to shift demand from risky to riskless assets, namely from equity to bonds and annuities. Then, as a consequence of the superiority of annuities over bonds, annuity demand increases due to health costs. For the reasons outlined in the introduction, we model annuitization as an irreversible decision at retirement and find that, in such a setting, it is optimal to annuitize fully so that people can save adequately out of the annuity income. The benefits of insurance against longevity risk and the mortality credit outweigh the reduction in both liquidity and the ability to get equity exposure at short horizons.

Note that we examine the fraction of *total* wealth that optimally should be annuitized, which consists of both pre-annuitized and non-annuitized wealth. The pre-annuitized wealth level mostly consists of social security income and private pensions. Given that (almost) full annuitization in real annuities is optimal, this can consist of for instance 80% pre-annuitized wealth and 20% liquid financial wealth but also of 50% pre-annuitized wealth and 50% liquid. But in any case (almost) all liquid wealth should be annuitized. In Section 3.6 we explore the optimal annuitization levels for varying pre-annuitized and liquid wealth levels where we assume that the pre-annuitized wealth level consists of inflation-indexed income. If in that case a real annuity is available, the results do not change, but just have to be interpreted differently. If the pre-annuitized fraction is 80% and the optimal total annuity level is 95%, then 75% of *liquid* wealth should be annuitized. If 50% is pre-annuitized this fraction is 90% of liquid wealth. However, this reasoning does not hold if on top of this pre-annuitized income only nominal annuities can be bought. Furthermore, the height of the total wealth level can have an effect on optimal annuity demand, because then the background risk and minimum consumption level are relatively higher or lower compared to total wealth.

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[8] As described in Section 2.3 we set, for ease of comparison, the initial wealth such that, if the individual would allocate his wealth fully to real annuities, the (real) income for the rest of the lifespan equals unity. We call this annuity income, the Full Real Annuity Income (FRAI).

a

Bequest motive: nominal annuity


b

Bequest motive: real annuity
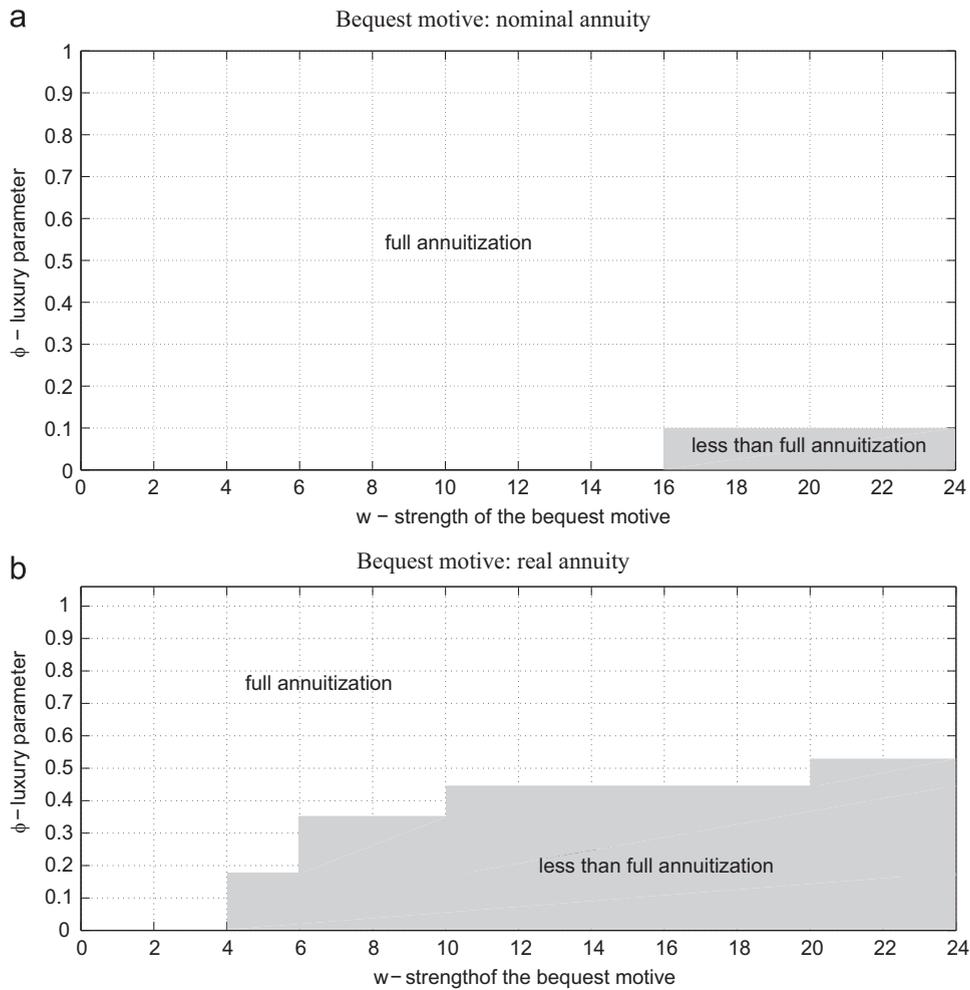
Fig. 3. Optimal annuitization for different parameters of the bequest motive Simulation results for the benchmark model including bequest motives. Panel (a) displays the bounds on the bequest motive parameters for which full annuitization holds, when an agent can invest in a nominal annuity. Panel (b) displays the bounds on the bequest motive parameters for which full annuitization holds, when an agent can invest in a real annuity. A higher strength of the bequest motive $\overline{w}$ gives incentives to annuitize less, while a higher luxury parameter ϕ gives incentives to annuitize more. The other parameters are those of the benchmark. All numbers are relative to the FRAI, which is the real annuity income if 100% is invested in a real annuity. Full annuitization is 95% or 100%, less than full annuitization is 90% or less.

3.2. Optimal annuitization strategies at retirement with bequest motives and default risk

The evidence in the previous section suggests that background risk and an incomplete annuity menu have at most only a small effect on optimal annuitization levels. However, there are other factors that might influence optimal annuitization behavior that we did not consider until now. Namely, retirees might want to leave a bequest for their heirs. However, if all wealth is annuitized it can be more difficult to leave a substantial bequest via saving out of the annuity income. Another reason to decrease annuity demand can potentially be default risk of the insurer. Especially due to the credit crisis, more consumers are aware of the risks of the annuity provider defaulting. In this section, we calculate bounds on a possible bequest motive and default risk of the insurer, such that our results still hold. We present results for two benchmark cases: an individual who can freely invest in a real annuity and someone who can freely invest in a nominal annuity. In all cases the other assumptions, including those on background risk, are as before, unless explicitly stated otherwise.

The desire to leave a bequest might induce agents to decrease annuity demand to have wealth liquid to leave as a bequest. However, the evidence on whether agents indeed have a bequest motive is mixed (Ameriks et al., 2011; Brown, 2001; Lockwood, 2012; Love et al., 2009). Following Ameriks et al. (2011) and De Nardi et al. (2010), we model the bequest motive as follows. An agent derives utility from leaving a bequest B_t:

$$v(B_t) = \frac{\overline{w}}{1-\gamma} \left(\phi + \frac{B_t}{\overline{w}} \right)^{1-\gamma}, \tag{19}$$

a

b
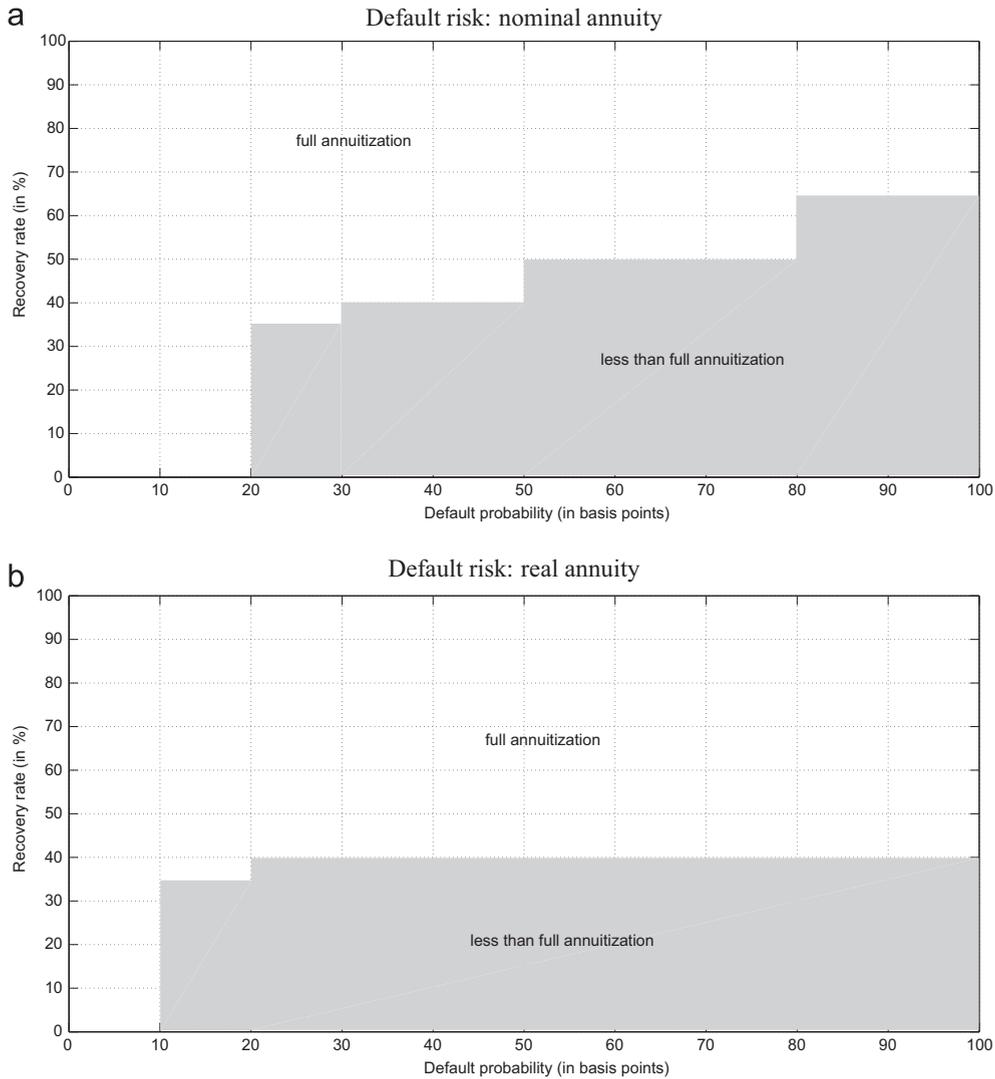
Fig. 4. Optimal annuitization with default risk Results for the benchmark model including default risk. Panel (a) displays the bounds on the default risk parameters for which full annuitization holds, when an agent can invest in a nominal annuity. Panel (b) displays the bounds on the default risk parameters for which full annuitization holds, when an agent can invest in a real annuity. A higher default probability gives incentives to annuitize less, while a higher guarantee gives incentives to annuitize more. The other parameters are those of the benchmark. Full annuitization is 95% or 100%, less than full annuitization is 90% or less.

where $\overline{w}$ is the strength of the bequest motive and ϕ is the prevalence in the population of an bequest motive. ϕ determines the curvature of the bequest motive and hence the extent to which bequests are a luxury good[9]. A bequest motive can give individuals incentives to annuitize less, because in case of early death, the retiree may not have had sufficient time to build up enough wealth to bequeath. If an agent dies at more advanced ages, the individual saves out of the annuity income to leave a bequest as before.

Fig. 3 shows for which values of the two bequest parameters ($\overline{w}$ and ϕ) full annuitization remains optimal. The effect of both parameters on the optimal annuity demand is in opposite directions. A higher strength of the bequest motive $\overline{w}$ gives an incentive to annuitize less, while a higher luxury good parameter ϕ increases the incentives to fully annuitize. Panel (a) presents the results in case only nominal annuities are available. We see that in almost all cases full annuitization

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[9] The optimal bequest in a simplified version of the model provides a better understanding of the meaning of the bequest parameters. In a riskless world the optimal solution can be obtained analytically: Assume an agent starts with an amount of wealth W, does not face longevity risk, and the time preference discount rate is zero. Each year the individual consumes C for T years and derives utility equal to $C^{1-\gamma}/(1-\gamma)$. At death, the retiree leaves a bequest B equal to $(W-CT)$ and derives utility from bequest equal to $(\overline{w}/(1-\gamma))(\phi+B_t\,\overline{w})^{1-\gamma}$. The agent chooses C optimally, to maximize total utility from consumption and the bequest. When differentiating total utility with respect to consumption, the resulting optimal consumption is $C=(W+\overline{w}\phi)/(\overline{w}+T)$ and the optimal bequest is $B=\overline{w}(C-\phi)$. Hence, the agent leaves a bequest to cover $\overline{w}$ years of spending for the heir at an annual expenditure level $(C-\phi)$, the amount by which his own optimal annual consumption exceeds the threshold ϕ. If W is too low to ensure an income stream for the heir higher than ϕ for $\overline{w}$ years, no bequest is left.

a

b

c
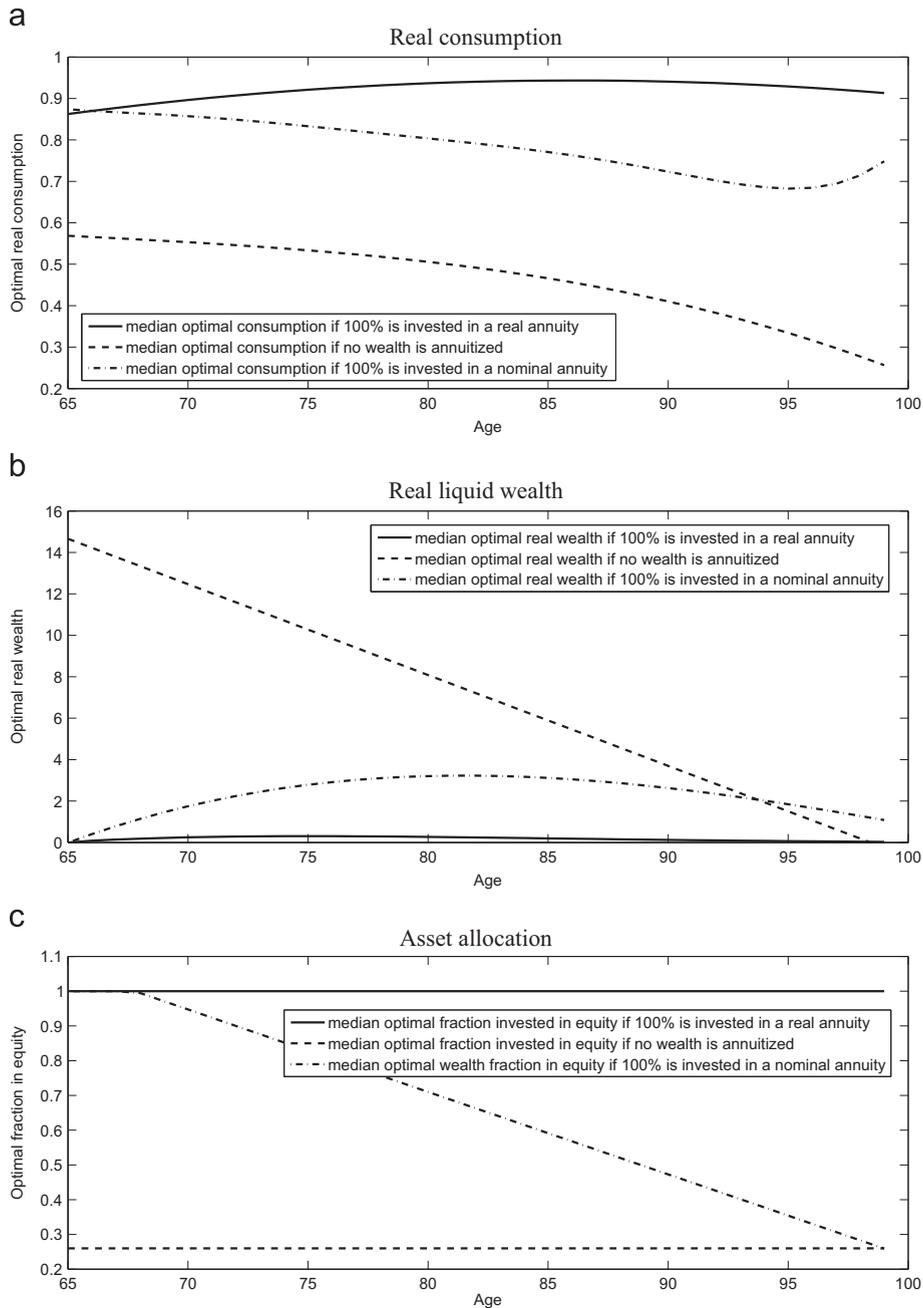
Fig. 5. Optimal real consumption, optimal real wealth, and optimal asset allocation Results for the benchmark model. Panel (a) displays the optimal real consumption for the optimal real annuitization level, optimal nominal annuitization level, and without annuities. Panel (b) displays the optimal *liquid* real wealth for the optimal real annuitization level, optimal nominal optimization level, and without annuities. Panel (c) presents the optimal fraction invested in the risky asset for the optimal real annuitization level, optimal nominal optimization level, and without annuities. Expenses due to background risk are included in the model. All numbers are in terms of the FRAI, which is the real annuity income if 100% is invested in a real annuity.

remains optimal. Only when the luxury parameter ϕ is 0.09 times the FRAI ($2000) or lower and the strength of the bequest motive ϕ is above 16, the optimal annuity demand falls. De Nardi et al. (2010) find a $\overline{w}$ (strength of the bequest motive) equal to 2.5 and Ameriks et al. (2011) estimate a $\overline{w}$ of 16 and a luxury parameters ϕ of 0.22 times the FRAI ($5000). For these parameters, full annuitization in a nominal annuity is optimal. It is optimal to annuitize your entire wealth and, subsequently, save to build up a buffer to leave as a bequest.

Panel (b) in Fig. 3 displays the bounds on the parameters for the bequest motive, when an agent can buy real annuities. In this case we see that for more values of the bequest parameters, full annuitization is sub-optimal. The reason is that the annuity income in the first years of retirement is higher for the nominal annuity than for the real annuity. The nominal annuity is front-loaded in real terms. For this reason the agent can build up a sufficient buffer faster when receiving a nominal annuity

income, to leave as a bequest in case of death already early in retirement. When comparing the bounds to the estimated parameters of Ameriks et al. (2011), we find that full annuitization is no longer optimal. In a riskless world with these parameters, the optimal consumption is about 0.51 FRAI ($11,500) and the optimal bequest is almost 4.6 times FRAI ($105,000). The agent reduces his annual consumption from 0.74 FRAI ($16,750) to 0.51 FRAI ($11,500) to leave this bequest.

Before, we implicitly assumed that the probability of default of the annuity provider is zero. However, a positive default probability can be another reason why agents might not want to annuitize fully. We assume that in case the annuity provider defaults, the agent recovers part of the present value of the annuity. First of all, after a default, part of the liabilities of the company can be covered. Second, if the amount recovered from the insurer is less than the guarantee of the state, then this amount is supplemented up to the guaranteed amount. In most states in the U.S. at least $100,000 is guaranteed and the maximum is $500,000 (Babbel and Merril, 2006). So even if the insurer goes bankrupt and the recovery value is low, the annuitant gets at least $100,000 of the present value of the annuity back. We assume that the agent gets a guarantee (free of default risk) from the state of $100,000, which is 4.4 times the FRAI.

In Fig. 4 we display the bounds on the default risk parameters for which full annuitization holds. The vertical axes in Fig. 4 specify which fraction is recovered in case of default. If this recovery rate times the present value of the annuity is less than 4.4 times the FRAI ($100,000) (and the present value of the annuity is more than $100,000), we assume the state guarantee supplements this amount up to 4.4 ($100,000). The horizontal axes display the default probability. First of all we see that the bounds differ substantially depending on the type of annuity the agent has; nominal or real. If an agent has a real annuity income, almost always full annuitization remains optimal while, if the individual can only invest in a nominal annuity, the optimal annuitization level potentially falls if the default probability gets high. The reason is that the nominal annuity is less welfare enhancing than the real annuity, hence there are more incentives to decrease the annuity level when default risk is high. Second, we see that the fraction of wealth recovered is important for the optimal annuity demand, which is similar to the findings of Babbel and Merril (2006). In this paper we normalized all numbers in terms of the Full Real Annuity Income, but the wealth that was used as a basis is $335,000, which is the median total wealth level at 65. Hence 33.5% of the value of the annuity at 65 is guaranteed by the state. Furthermore Moody's reports that the default probabilities, for corporates up to a rating of Baa, are about 16 bp.

We have determined that neither bequest motives nor default risk influence optimal annuity demand to a large extent. In the subsequent sections we will assume that both bequest motives and default risk are zero.

3.3. Consumption, wealth, and asset allocation paths over the life cycle

The optimal consumption and wealth trajectories including the asset allocation rules are presented in Fig. 5. This figure shows the median consumption, wealth, and asset allocation for three cases: (1) no annuitization, (2) 100% investment in nominal annuities, and (3) 100% investment in real annuities. Expenses due to background risk are included in this analysis.

Fig. 5a shows that, in case (1) and (2), the optimal consumption path is decreasing over time. This reflects the fact that if the longevity risk in the real consumption level is not hedged, agents do not plan much consumption at ages where the probability is high that one will have passed away. If real annuities are used, inflation risk can be hedged and the planned consumption path is
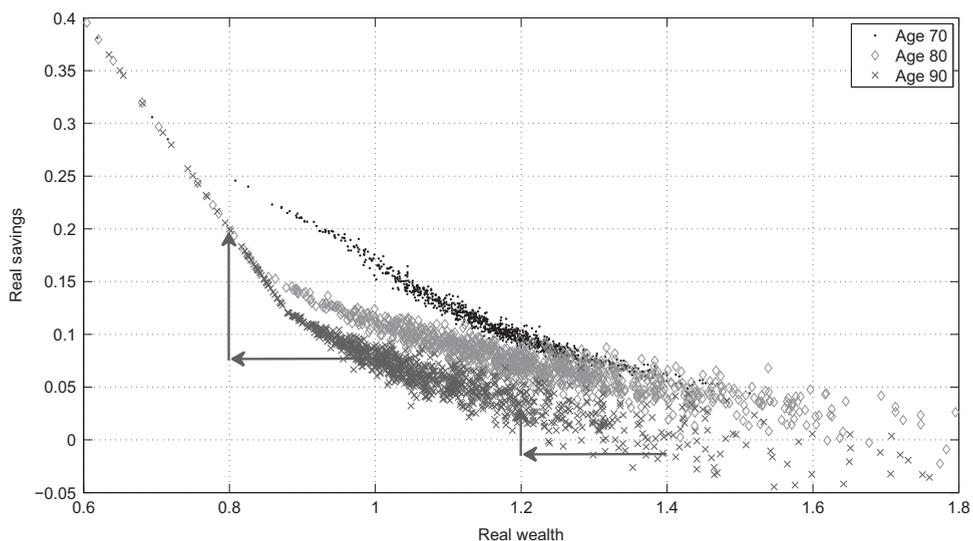
Fig. 6. Optimal savings for varying wealth levels when 100% is allocated to a real annuity results for the benchmark model. This figure shows the optimal real savings for varying levels of liquid real wealth if an agent invested his entire wealth in a *real* annuity. We show the real savings for the ages 70, 80, and 90. All numbers are in terms of the FRAI, which is the real annuity income if 100% is invested in a real annuity.
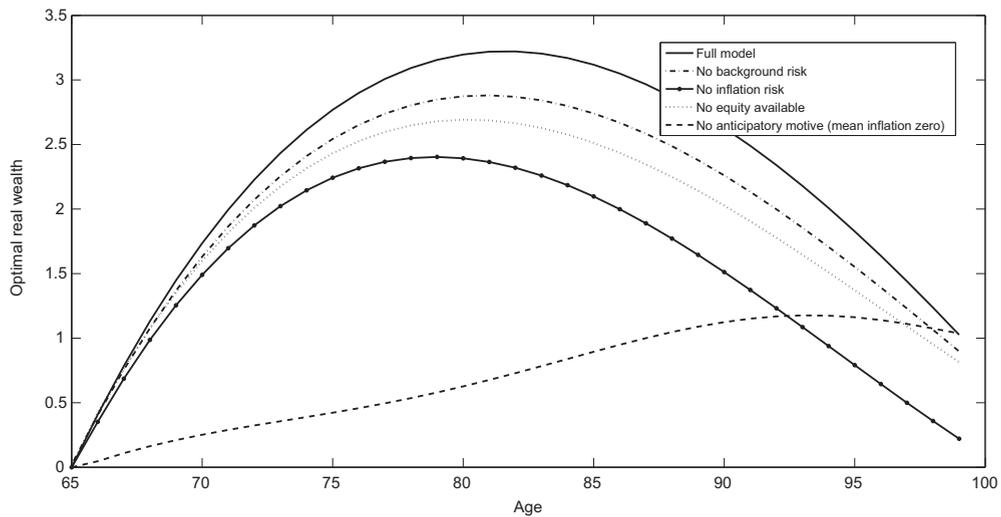
Fig. 7. Optimal real wealth trajectories when 100% is allocated to a nominal annuity Results for the benchmark model. This figure shows the optimal liquid real wealth trajectories for five variations of the parameter values. These are the wealth paths for an agent who invested his entire wealth at 65 in this *nominal* annuity. The liquid wealth trajectories are for the case where 100% is invested in a nominal annuity. In the model setup where inflation risk is excluded, the inflation level is fixed at 3.48%. All numbers are in terms of the FRAI, which is the real annuity income if 100% is invested in a real annuity. We set this Full Real Annuity Income equal to unity.

approximately flat (in real terms) because of the fact that the time-preference parameter and interest rates coincide approximately. Early in retirement, consumption is reduced to build up a buffer against expenses due to background risk.

Fig. 5b displays that only a relatively small amount of liquid wealth is accumulated if real annuities are available. That level of liquid wealth is sufficient to cover for unexpected shocks (in background risk), but there are no anticipatory savings due to inflation needed. The median liquid wealth trajectory is very different if nominal annuities are used. In that case the individual saves substantially out of the nominal annuity income and a median real wealth of 3.2 times the FRAI is attained at the age of 80. This liquid capital is needed to have sufficient *real* consumption if the agent lives to an advanced age. This is in accordance with Love and Perozek (2007), who find that background risk increases the optimal amount of liquid assets.

Panel C of Fig. 5 shows that the optimal fraction of liquid wealth invested in the risky asset, if a person has annuitized nothing, is about 26% and is fixed over time. Instead the optimal fraction is 100% if an individual has invested optimally in a real annuity. If a person invests all wealth in a nominal annuity at age 65, we find that the optimal fraction depends negatively on the fraction of liquid wealth compared to total wealth (liquid wealth plus discounted value of annuity income). This result is in line with Cocco et al. (2005).

3.4. Saving out of real annuity income

We find that full annuitization remains optimal if agents save, and invest, optimally out of their annuity income. In this section we examine this savings mechanism further. Fig. 6 displays the optimal real savings out of the real annuity income for varying real wealth levels, at the ages 70, 80, and 90. If an agent has a wealth level of 1 times the FRAI and is 90 years old (crosses), savings are about 0.08 times the FRAI. Put differently, the individual saves 8% out of his real annuity income to increase his buffer. So even if an agent is 90 years old, if the buffer is insufficient, savings are positive to increase it. Furthermore we see that the amount of savings decreases with age, for a given wealth level. For a wealth level of one FRAI, the real savings are 17% for a 70 year old, 11% for a 80-year old, and 8% for a 90-year old.

From Fig. 6 we can also derive the effect of background risk on the amount of savings, which is illustrated by the arrows. Consider a 90-year old agent with liquid wealth equal to one FRAI. If this agent is hit by background risk and needs to pay expenses equal to 0.2 times the FRAI, his wealth drops from 1 to 0.8 (left horizontal arrow). As a reaction to this, the individual increases savings from 8% of his annuity income to 20%. This increase in savings is substantial, because the buffer that the retiree started with was not that high. If the agent has more wealth, the reaction is less if the retiree is hit by the same background risk expenses, because the buffer is already high. This can be seen from the arrows on the right, the speed with which the buffer is rebuild falls with the wealth level. As a side effect, the figures illustrate the saving behavior of those with low wealth. A 90-year old with a real annuity income and wealth less than 1.2 times the FRAI should still save to hedge against background and inflation risk.

3.5. Saving out of nominal annuity income

Fig. 7 analyzes in more detail the most striking result of Fig. 5: the capital accumulation in case of nominal annuitization. Individuals save out of nominal annuity income for four different reasons. A first reason is real consumption smoothing,
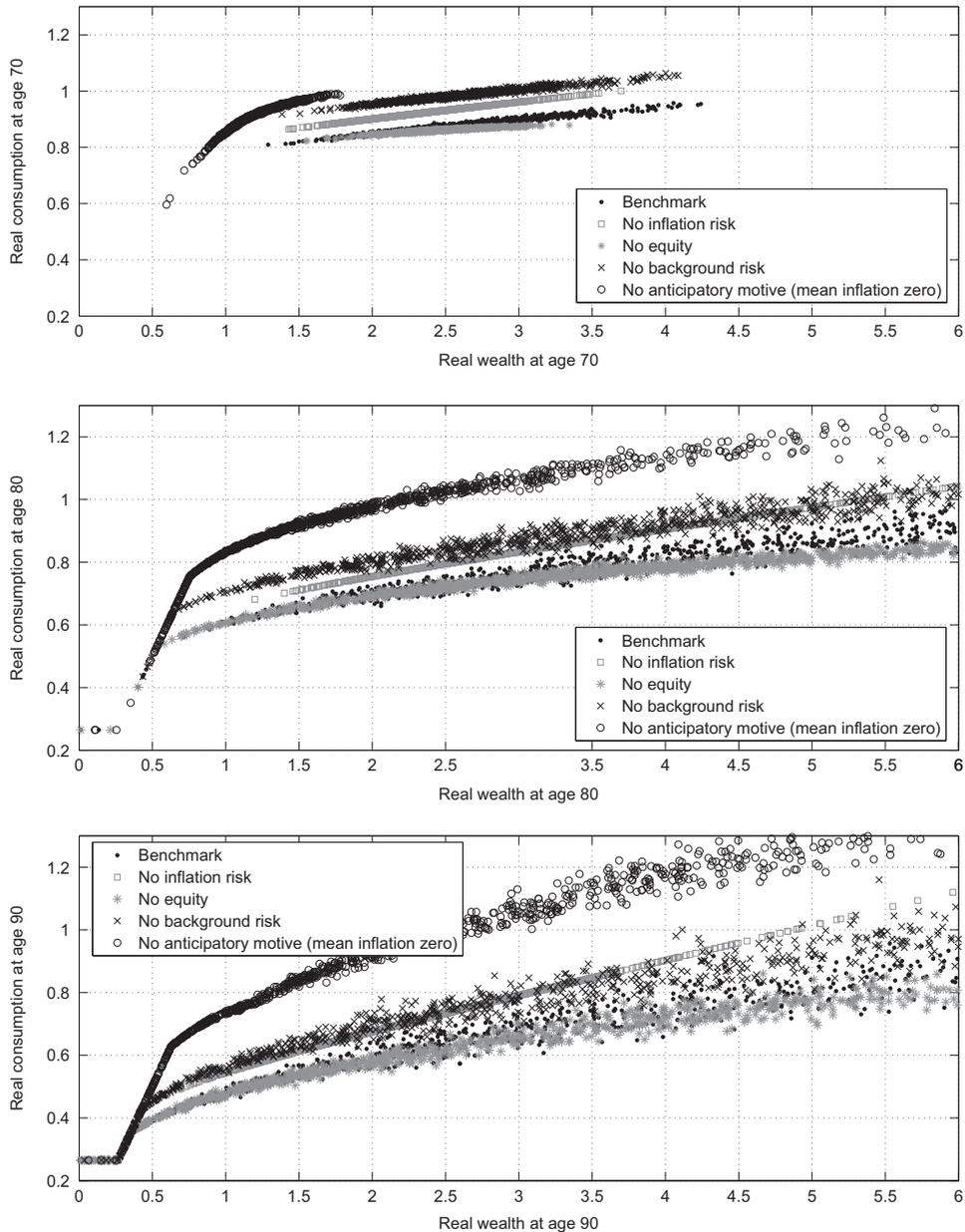
Fig. 8. Optimal consumption for varying wealth levels when 100% is allocated to a nominal annuity Results for the benchmark model. The above panel displays the optimal real consumption for a 70 year old for several liquid real wealth levels. These consumption/wealth strategies for an agent who invested is entire wealth at 65 in a *nominal* annuity. The liquid real wealth levels are after annuity income and expenses due to background risk. Hence the real wealth level is the disposable wealth level. The middle panel shows the optimal real consumption levels per real wealth level for a 80 year old and the lower panel for 90 year old. The parameters are that of the benchmark set up. All numbers are in terms of the FRAI, which is the real annuity income if 100% is invested in a real annuity. We set this Full Real Annuity Income equal to unity.

because even deterministic inflation erodes the real consumption that can be obtained from nominal annuity income. A second reason relates to inflation *risk*. Inflation risk generates precautionary savings as inflation risk can be seen in this setting as a (partly) unhedgeable background risk. The third reason is precautionary saving to hedge for background risk. The final motivation is to accumulate capital to capture the equity risk premium.

Fig. 7 presents the optimal median wealth path for five different specifications of the model to disentangle the different reasons for capital accumulation mentioned above. The solid line is the median wealth path for the full model, which is the same as we displayed in Fig. 5b. Its maximum value is about 3.2 FRAI at age 82. To disentangle these four effects, we remove each motive for savings separately. We examine the effect of anticipating an average inflation of 3.48%, by setting the mean inflation equal to zero while keeping the standard deviation of the instantaneous inflation rate equal to 1.38% and by reducing the nominal interest rate and the equity return by 3.48%. If no deterministic inflation is incorporated (dashed line)

Table 1

Optimal *real* annuitization levels (%) for varying pre-annuitized and liquid wealth levels results for the benchmark model. This table reports the optimal annuity levels (in %) in a *real* annuity. The pre-annuitized wealth level is an inflation-indexed pension income. The number without brackets is the optimal annuity demand as a fraction of total wealth and the number between brackets is the optimal annuity demand as a fraction of liquid financial wealth. For instance, if the pre-annuitized wealth is 6.6 and liquid wealth is 2.2, then 75% of total wealth is pre-annuitized. If then the optimal annuity level is 93%, this means that the optimal annuity demand as a percentage of liquid wealth is 72%. All numbers are relative to the FRAI, which is the real annuity income if 100% is invested in a real annuity. The rest of the parameters are as in the benchmark case.

Pre-annuitized wealth	Financial wealth					
	2.2 ($50k)	4.4 ($100k)	6.6 ($150k)	8.8 ($200k)	11 ($250k)	13.2 ($300k)
6.6 ($150k)	93	94	94	95	96	97
	(72)	(85)	(88)	(91)	(94)	(96)
8.8 ($200k)	94	94	95	96	97	97
	(70)	(82)	(88)	(92)	(95)	(95)
11 ($250k)	94	95	96	97	97	97
	(65)	(83)	(89)	(93)	(94)	(95)
13.2 ($300k)	95	96	97	97	97	97
	(64)	(84)	(91)	(93)	(93)	(94)

the maximum amount of wealth accumulated drops to 1.2 times the FRAI[10]. Hence the largest part of the saving is due to the first motive: agents shift (nominal) income from early in retirement to later. Furthermore we see that the shape of the path of wealth differs substantially. The reason is that if the mean inflation is zero, agents do not need to accumulate large amounts of wealth in the beginning of retirement and dissave at later ages, to be able to have a smooth consumption pattern over the life. They only need a buffer (against background and inflation risk), and, at the same time, use it to get equity exposure. This buffer is accumulated gradually over time to smooth consumption. Hence, the *level* of inflation explains a substantial part of the results, but the other three factors also induce savings.

In order to examine the effect of inflation *risk*, we set the standard deviation of the instantaneous inflation rate and the standard deviation of the price index to zero. The optimal maximum savings amount decreases with some 25% if inflation risk is taken out (from 3.2 times the FRAI to 2.4 times the FRAI). The level of precautionary savings is enhanced by the persistence in inflation.

The median savings is reduced from approximately 3.2 times the FRAI, if all risk factors are included, to 2.7 times the full real annuity income if agents cannot invest in equity. We calculate this effect by assuming that agents can only invest in a 1-year nominal bond. Hence savings are increased substantially to be able to profit from the equity risk premium.

If we assume that agents do not face background risk, the amount of savings is about 2.9 times the FRAI. Similarly, Palumbo (1999) and De Nardi et al. (2010) find that uncertain medical expenses increases the amount of precautionary savings. In sum, an individual could also simply annuitize less to keep wealth liquid and extract wealth from the savings account to insure against inflation shocks. However, we find that instead it is optimal to annuitize fully (and receive the mortality credit) and, subsequently, save out of the annuity income.

The previous paragraph shows different median wealth paths for an agent who invests everything in a nominal annuity. However, it is also interesting to consider consumption/savings strategies for wealth levels above or below the median. Fig. 8 displays the optimal consumption for various wealth levels at age 70, 80, and 90 and for the different risk factors that the agent faces. Note that in Fig. 6, we display the real savings on the y-axis, while in Fig. 8 we display real consumption. The dots are for the benchmark specification, hence agents save due to deterministic inflation, inflation risk, background risk, and to get equity exposure. The real wealth level displayed on the horizontal axes is the remaining wealth *after* the agent payed his expenses due to background risk and received the annuity income. If we look at the upper panel for a 70-year old, we see that, for the benchmark case, if an agent has a wealth level of 1.5 times the FRAI, consumption is equal to 0.82 times the FRAI. Furthermore, we see that the consumption increases in the wealth level. If an agent cannot invest in equity, consumption is similar for wealth levels below twice the FRAI, but less for higher wealth levels. The reason for the lower consumption level is that the agent wants to have a larger amount of liquid wealth to invest in equity. When we compare the real consumption levels when an agent does not face background risk (crosses) with the benchmark case, we see that the consumption level is lower due to the background risk. Furthermore we see that the real consumption level is reduced less due to inflation risk. If there is no inflation risk (squares), agents with a wealth level of 1.5 consume about 0.87, compared to 0.82 when individuals do face inflation risk. However, if agents do not have an anticipatory motive to save (circles), they increase consumption levels substantially. Moreover these patterns of differences in real consumption, for different specifications of the model is similar for a 70, 80, or 90 year-old. This can be seen by comparing the three panels of Fig. 8.

The middle panel of Fig. 8 shows the optimal real consumption for 80-year olds and the lower panel for a 90-year old. There are several things apparent from these graphs. First of all, we see that, if the real wealth level is low, agents consume their entire wealth. For instance, if the wealth level of a 80-year old is about 0.5 times the FRAI, the individual consumes this entire amount. Second, when comparing the three panels, we see that for a liquid real wealth level of 1.5 FRAI, the real

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[10] Note that this optimal wealth path is equal to the optimal wealth path when an agent receives an annuity income which is increasing with the expected mean annual inflation. In several countries these increasing annuities are available, but not sold that often.

Table 2

Optimal *nominal* annuitization levels (%) for varying pre-annuitized and liquid wealth levels Results for the benchmark model. This table reports the optimal annuity levels (in %) in a *nominal* annuity. The pre-annuitized wealth level is an inflation-indexed pension income. The number without brackets is the optimal annuity demand as a fraction of total wealth and the number between brackets is the optimal annuity demand as a fraction of liquid financial wealth. For instance, if the pre-annuitized wealth is 6.6 and liquid wealth is 2.2, then 75% of total wealth is pre-annuitized. If then the optimal annuity level is 97%, this means that the optimal annuity demand as a percentage of liquid wealth is 88%. Note that the optimal annuity percentage as a fraction of total wealth is for the combination of both the real annuity (pre-annuitized wealth) and the nominal annuity (liquid wealth). All numbers are relative to the FRAI, which is the real annuity income if 100% is invested in a real annuity. The rest of the parameters are as in the benchmark case.

Pre-annuitized wealth	Financial wealth					
	2.2 ($50k)	4.4 ($100k)	6.6 ($150k)	8.8 ($200k)	11 ($250k)	13.2 ($300k)
6.6 ($150k)	97	97	100	100	100	100
	(88)	(91)	(100)	(100)	(100)	(100)
8.8 ($200k)	97	99	100	100	100	100
	(85)	(97)	(100)	(100)	(100)	(100)
11 ($250k)	97	99	100	100	100	100
	(82)	(97)	(100)	(100)	(100)	(100)
13.2 ($300k)	97	99	100	100	100	100
	(75)	(93)	(100)	(100)	(100)	(100)

Table 3

Welfare gains (in %) of investing the optimal amount in a combination of variable and real annuities compared to only real annuities Results for the benchmark model. The assumed interest rate (AIR) is either 4% or 2%. The rest of the parameters are as in the benchmark case.

	AIR 4%	AIR 2%
Background risk included		
Welfare gain	1.2%	1.1%
Optimal real/variable annuity	90/10	90/10
Background risk excluded		
Welfare gain	1.5%	1.3%
Optimal real/variable annuity	85/15	85/15

consumption depends negatively on age. The reason is that the nominal income in real terms decreases over time and the desired real consumption level falls because agents discount the future more heavily due to the probability of dying.

3.6. Optimal annuity levels for varying pre-annuitized wealth and liquid wealth levels

In the previous sections, we showed the optimal annuitization, consumption, and savings levels for the benchmark total wealth level at age 65 which is 15 times the FRAI (this is equivalent to $335,000). We displayed the optimal annuity demand as a fraction of the *total* wealth level, because most papers present the empirical annuity levels in this way (Dushi and Webb, 2004 and Pang and Warshawsky, 2010). We find that (almost) full annuitization of total wealth is optimal, where total wealth consists of pre-annuitized pension wealth (social security and defined benefit pension wealth) and liquid financial wealth. However, we did not show optimal annuitization as a fraction of liquid wealth, which off course is lower than that of total wealth if some wealth is already pre-annuitized. In Table 1 we present the optimal annuitization fractions of total and liquid wealth for varying liquid wealth and pre-annuitized wealth levels. The pre-annuitized wealth is assumed to be an inflation-indexed (pension) income and the agent optimally chooses which part of his financial wealth to invest in a real annuity. As a result, the optimal annuity levels are presented in two different formats: the numbers without brackets display the (1) optimal percentage in annuities as a fraction of *total* wealth and the numbers between brackets show the (2) optimal percentage in annuities as a fraction of *liquid* wealth. First of all, we see that the optimal annuitization level as a fraction of total wealth is almost 100%. Naturally, when displayed as a percentage of liquid financial wealth it is lower, but still the annuity levels are much higher than found in reality. The empirical distribution of wealth for the 2nd income decile is a bit more than twice the FRAI ($50,000) of financial wealth and 6.6 ($150,000) in pre-annuitized wealth[11]. As we can see in Table 1, these retirees optimally invest 72% of their liquid wealth in annuities, on top of the 75% of wealth that is already annuitized. Hence the optimal annuitization level as a percentage of total wealth is 93%. The liquid wealth level and pre-annuitized wealth level for an agent with a median income is 6.6 ($150,000) and 8.8 ($200,000) respectively. This agent optimally annuitizes 88% of his liquid wealth. In Table 2 we show the optimal annuity levels when agents can invest their liquid financial wealth only in nominal annuities. Note that now the optimal annuitization level of total wealth consists of both a real annuity (pre-annuitized

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[11] We use the numbers from Pang and Warshawsky (2010), who calculate these levels on a household basis which we divide by the average household size.

Table 4

Robustness tests The table reports the optimal annuitization levels (in %) for several alternative parameter choices. For every robustness check one parameter is changed and the rest remains at their benchmark value.

Parameter setup	Optimal level Real annuities	Optimal level Nominal annuities
Benchmark parameters	96	100
Mean gross equity return 10% instead of 8%	93	100
Mean gross equity return 12% instead of 8%	88	91
Mean gross equity return 14% instead of 8%	79	79
Mean gross equity return 16% instead of 8%	70	67
Mean gross equity return 18% instead of 8%	60	55
Mean gross equity return 20% instead of 8%	50	42
Background risk persistent	95	100
Mean expenses due to background risk 0.2 instead of 0.1 FRAI	91	100
Mean expenses due to background risk 0.3 instead of 0.1 FRAI	97	100
Volatility background risk 0.14 instead of 0.07 FRAI	85	97
Subsistence consumption level 0.5 instead of 0.25 FRAI	94	100
Female survival probabilities instead of male	93	100
Expense factor 7.3% instead of 0%	94	100
Expense factor 15% instead of 0%	92	100
Risk aversion coefficient 2 instead of 5	92	100
Risk aversion coefficient 10 instead of 5	96	94

wealth) and a nominal annuity (liquid wealth). In this case we find similar results: the optimal annuity levels are very high and incomplete annuity markets and background risk do not explain the annuity puzzle.

Furthermore, we see that when the total wealth level is lower, the optimal annuity demand is a bit lower as well. The reason is that an agent with a lower wealth level needs to keep a larger fraction liquid at the beginning of retirement to have the same absolute buffer against background risk. In addition, the optimal annuity demand differs depending on whether the agent can invest in a nominal or a real annuity, on top of his inflation-indexed pension income. The optimal annuity level as a fraction of total wealth when an agent has 6.6 pre-annuitized and 2.2 liquid wealth is 93% if he has a real annuity available and 97% if he can only invest in a nominal annuity. The nominal annuity is more attractive, because it generates a higher real income early in retirement so that the agent can save quicker to have a sufficient buffer against background risk shocks. Hence the agent needs to reduce the optimal annuity demand less compared to the real annuity.

3.7. Welfare gains of variable annuities

The literature has examined welfare gains due to variable annuities (see, e.g., Koijen et al., 2011, and Horneff et al., 2008b). This section examines whether adding variable annuities to the menu increases welfare sizeably in our setup with post-retirement savings. Table 3 displays the welfare gains from allocating the optimal amount to a variable and a real annuity, compared to only a real annuity. We see that the welfare gains are at most 1.5%. Hence, adding a variable annuity to the menu does not lead to a large increase of welfare if agents save out of their annuity income to invest in equity. The combined optimal annuity portfolio for an individual who faces background risk is only 10% in a variable annuity, with the remaining wealth in a real annuity. The reason is that individuals can save out of their annuity income to get equity exposure and real annuities provide a much better hedge against inflation risk than equity-linked annuities.

Koijen et al. (2011) find an optimal allocation of 40% to variable annuities. However, they do not include equity in the post-retirement asset menu. Hence, the only way in which agents can get equity exposure, is via a variable annuity. For this reason the welfare gains that they find are much higher than ours. Similar reasoning holds for the contrasting results with Brown et al. (1999). Horneff et al. (2009) find a welfare gain of 6% at age 80 and 30% at age 40 of investing in variable annuities instead of nominal annuities. They, however, assume that the asset allocation of the portfolio linked to the variable annuity can vary over time and additional annuities can be bought every year. This strand of literature includes annuities in the asset allocation menu, and agents decide how much to invest in equity/bonds/annuities annually (Horneff et al., 2008b). In that case agents do not fully annuitize at age 65, to invest in equity. As agents get older they gradually invest all their wealth in annuities, as they become more attractive than equity due to the mortality credit.

4. Robustness tests

As a first robustness check, we increase the equity premium to an expected stock return of 10% rather than 8%. Not surprisingly, this implies a reduction in annuity demand, but the numerical effect is small. The optimal demand for real annuities reduces from 96% to 93%. For the nominal annuity case, full optimization remains optimal. For higher levels of expected stock returns, the optimal annuitization level is lower. If the gross equity return is 20%, the optimal annuitization

levels are around 50%. The reason is that agent's are comparing the benefit of the mortality credit to the return on equity. Hence a higher return on equity deters from annuitizing fully.

The background risk in our benchmark case consists of i.i.d. shocks, while, as a robustness test, we assume the risk follows an autoregressive process with an AR(1) coefficient of 0.9. The unconditional mean and standard deviation of the expenses due to background risk are the same as in the benchmark case. We find that, even if the process of the background risk is highly persistent, high annuity levels are still optimal. As another check for robustness the mean (real) expenses due to background risk are doubled from 10% to 20% of the full real annuity income. Moreover the standard deviation is doubled as well. The optimal level allocated to a real annuity decreases from 96% to 91%. Again the direction of the effect is as expected, but the numerical differences are small

As a subsequent test, we double the subsistence consumption level to examine whether this alters the optimal level, Table 4 shows that this is hardly the case. Similary when we assume female survival probabilities instead of male. In addition we consider the effect on optimal annuitization of including a load factor on the annuity income. The load factor is set at 7.3% in line with Mitchell et al. (1999). The optimal annuitization level falls by only 2%. Naturally the welfare loss of the load is large, 8.5%[12]. Finally a less risk averse individual ($\gamma = 2$) invests 92% of his initial wealth in real annuities. Thus the change in the optimal annuitization level is quantitatively small and the previous results are also robust for an alternative risk preference.

5. Conclusion

This paper analyzes whether optimal annuity demand is affected by incomplete annuity markets, background risk, bequest motives, and default risk. If no variable annuities are available and borrowing constraints are imposed, it can potentially be optimal to annuitize only a part of your wealth. However, we find that (almost) full annuitization remains optimal, irrespective of whether nominal or real annuities are available if agents can save adequately out of their annuity income. In case of nominal annuities, the agent will save considerably out of the annuity income during retirement to gain equity exposure and hedge against background and inflation risk. If an individual receives a real annuity income, the agent saves a smaller amount as a buffer against (real) background risk. In all cases (close to) full annuitization at age 65 remains optimal. As a side result, we find that access to variable annuities is less welfare enhancing than previously found in the literature. The argument is similar: the buffer saved can be used to get sufficient equity exposure.

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[12] This result is not presented in the paper. The percentage welfare loss is larger than the load, because the amount of income after paying the expenses due to background risk falls by a larger percentage than the load. The income available for consumption does not scale down by the load percentage, due to the expenses for background risk.

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